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ELECTRIC MONOPOLE TRANSITIONS AND BETA BANDS IN EVEN NUCLEI

by

J. P. Davidson*

U.S. NAVAL RADIOLOGICAL
DEFENSE LABORATORY

SAN FRANCISCO · CALIFORNIA · 94135

NUCLEAR RADIATION PHYSICS BRANCH
J. M. Ferguson, Head

NUCLEONICS DIVISION
C. S. Cook, Head

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*Visiting Scientist-in-Residence, on leave of absence from Rensselaer Polytechnic Institute, Troy, N. Y., academic year 1964-65.

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Eugene P. Cooper

Eugene P. Cooper
Scientific Director

D. C. Campbell, CAPT USN
Commanding Officer and Director

ABSTRACT

Reduced transition probabilities for electric monopole transitions ρ^2 have been calculated using a collective model with deformation vibrations but no asymmetry vibrations. Both ρ^2 and $X = \rho^2 R_o^4/B(E2)$ have been evaluated within the context of the model by exact numerical methods for transitions within positive as well as negative parity rotational bands of even nuclei. Comparison of experiment with theory shows quite good agreement in the actinide deformed region; however, for the rare earths theoretical predictions are generally an order of magnitude greater than the measured values.

SUMMARY

Recent exact numerical calculations of the influence of deformation vibrations on electric quadrupole transitions in deformed even nuclei have been extended to a study of electric monopole transitions in such nuclei. These monopole transitions can provide an insight into the nature of the excited 0+ states, in particular into the excited K=0 beta bands. Comparison with experiment shows that model predictions are adequate for the very heaviest nuclei but are far too great for nuclei in the rare earth region.

1. Introduction

Recent investigations of the low-lying rotational levels in even nuclei have shown that the ground state rotational band structure can be accurately reproduced[1,2]) by using a very simple collective model first introduced by Davydov and Chaban[3]). It was also shown that if the stiffness parameter μ was determined from the energy level spacings in the ground state rotational band then the beta (K=0) band head could be predicted to within 20%, and often considerably less[1]). The consistency of this model has been checked by calculating exactly the effect beta vibrations have on the electric quadrupole transitions in such nuclei[4]). The results of this calculation showed by properly including these beta vibrations the E2 branching ratios compare well with experiment. In particular, in the coulomb excitation of the beta band I=2 level it was found that the theory is within experimental error of the measured values (recent unpublished data on ^{154}Sm gives a value of about half that predicted by theory[5])). Agreement between theory and experiment has also been found to be quite satisfactory for the coulomb excitation of the so-called gamma band I=2 level[6]). It would seem that this simple collective model is capable of quite accurate predictions not only of energy levels but also of B(E2)

branching ratios. With increasingly more experimental data available on electric monopole transitions it appears useful to compare this type of data with the theory. Indeed, Davydov and co-workers have done just that[7,8]; however, their work includes the effects of gamma or asymmetry vibrations and is therefore necessarily approximate in nature. It is the purpose of this paper to treat the effect of deformation vibrations on E0 transitions exactly while at the same time not requiring axial symmetry. The results of refs. [1], [2], and [4] show that asymmetry vibrations can be treated as a small perturbation. In general the gamma band mixing into the ground state and beta bands is small and a perturbation treatment of gamma vibrations is probably adequate except where rather strong monopole transitions occur between gamma and ground state bands. At present there seems only one nuclide where this occurs and that is in ^{232}Th. Here the I=2 levels of the beta and gamma bands are only 11 keV apart and a relatively strong E0 transition from the gamma band I=2 level to the ground state band has been observed[9].

In the remaining part of this section an outline of the vibrational problem will be given in order to fix the notation. In Section 2 the electric monopole transition probabilities will be derived. Finally, in Section 3 we compare experiment with theory.

In what follows we shall use a generalized treatment of deformation vibrations[10] equally applicable to quadrupole vibrations and octupole vibrations. While no E0 transitions have been definitely observed between negative parity levels they should in principle be no different

from such transitions observed between appropriate positive parity levels. This treatment is essentially the usual one for quadrupole surfaces[3]) (it should be noted that a diagonal term omitted in the vibrational Hamiltonian of ref. [3]) is included here and has a significant effect on the eigenvalues of the I=0 and 2 levels[15])). For octupole surfaces the condition which diagonalizes the momental ellipsoid is used[10]), so that no terms with K=1 or 3 appear in the state functions. This is consistent with a recent microscopic calculation of Soloviev and co-workers[11]) in which they find that the $\lambda=3$, $|\mu| = 1$ and 3 degrees of freedom do not possess very collective properties.

As usual one begins by expanding the nuclear surface in the laboratory coordinate system

$$R(\theta,\phi) = R_o\left[\alpha_o + \Sigma_\mu \, \alpha_{\lambda\mu}{}^* \, Y_{\lambda\mu}(\theta,\phi)\right] , \qquad (1)$$

here $\lambda=2$ for positive parity and $\lambda=3$ for negative parity states while α_o is unity to first order and the second order differences are usually neglected. It is known, however, that these volume conserving second order terms are important in electric monopole calculations[12]) and they will be retained here. Small oscillation theory then yields the classical Hamiltonian

$$H_\lambda = \frac{1}{2} B_\lambda \, \Sigma_\mu \, |\dot{\alpha}_{\lambda\mu}|^2 + \frac{1}{2} C_\lambda \, \Sigma_\mu \, |\alpha_{\lambda\mu}|^2 , \qquad (2)$$

$$H = \Sigma_\lambda H_\lambda.$$

For vibrational nuclei this can be quantized straight away and, using the number representation $|N>$, one can easily show that E0 transitions are allowed between any two states of the same spin for which ΔN=2. This selection rule then prohibits such transitions between the one and two phonon I=2+ states. Making use of the transformation

$$\alpha_{\lambda\mu} = \Sigma_\upsilon D_{\mu\upsilon}^{\lambda*}(\theta_i)\, \alpha_{\lambda\upsilon} \tag{3}$$

where the $D_{\mu\upsilon}^{\lambda}$ are the (2I+1)-dimensional representation of the rotation group[13]) and the θ_i are the Euler angles relating laboratory and body-fixed axis systems. It is useful if the body expansion coefficients in (3) are parameterized as[10])

$$a_{\lambda\mu} = \beta_\lambda\, \sigma_{\lambda\mu} \tag{4}$$

where β_λ are the λ^{th}-order deformation parameters while the asymmetry parameters $\sigma_{\lambda\mu}$ may be subjected to the additional condition

$$\Sigma_\mu\, \sigma_{\lambda\mu}^{\;2} = 1. \tag{5}$$

The expansion of the nuclear radius in the body-fixed system becomes

$$R(\theta', \phi') = R_o \left[1 - \frac{1}{12\pi} (3\beta_\lambda^2 + \beta_\lambda^3 T_\lambda) + \beta_\lambda \Sigma_\mu \sigma_{\lambda\mu} Y_{\lambda\mu}(\theta', \phi') \right]. \tag{6}$$

The quantity T_λ, which arises from the condition of volume conservation, is

$$T_\lambda = \sqrt{\frac{2\lambda+1}{4\pi}}\ C(\lambda\lambda\lambda;000)\ \Sigma_{\mu\mu'}\ \sigma_{\lambda\mu}\ \sigma_{\lambda\mu'}\ \sigma_{\lambda\mu-\mu'} \times C(\lambda\lambda\lambda;\mu-\mu',\mu',\mu) \tag{7a}$$

which for deformed nuclei will give rise to E0 transitions between gamma-like and ground bands. From the properties of the Clebsch-Gordan coefficients $C(L_1,L_2,L_3;m_1,m_2,m_3)$ it is seen that T_λ vanishes for λ odd. Thus, in principle this model permits no monopole transitions between negative parity states except between zeta and ground bands. For positive parity states on the other hand transitions between gamma and ground state bands are possible. By expressing the asymmetry parameters in the familiar form

$$\sigma_{20} = \cos\gamma,\ \sigma_{2\pm2} = \frac{1}{\sqrt{2}} \sin\gamma,$$

$$\sigma_{30} = \cos\eta,\ \sigma_{3\pm2} = \frac{1}{\sqrt{2}} \sin\eta$$

T_λ can be expressed as[7])

$$T_\lambda = \frac{1}{7}\sqrt{\frac{5}{\pi}}\cos 3\gamma \qquad \lambda=2$$
$$= 0 \qquad \lambda=3 \tag{7b}$$

The Hamiltonian of the system is obtained from eq. (2) by using relation (3) and recalling that since the deformation vibrations are to be treated exactly and the asymmetry vibrations only in perturbation theory the space with respect to which the quantization is carried out contains but four dimensions: β_λ and the three Euler angles θ_i. The $\sigma_{\lambda\mu}$ are taken as fixed. The Schrödinger equation separates into rotational and vibrational parts.

$$\left[\frac{1}{2}\sum_k (I_k^2/i_k^\lambda) - \epsilon_{IN}^\lambda\right]\psi_{IN}(\theta_i) = 0 \tag{8}$$

$$\left[-\frac{\hbar^2}{2B_\lambda}\frac{1}{\beta_\lambda^3}\frac{d}{d\beta}\left(\beta_\lambda^3\frac{d}{d\beta_\lambda}\right) + \frac{\hbar^2}{4B_\lambda\beta_\lambda{}^2}\epsilon_{IN}^\lambda + \frac{1}{2}C_\lambda(\beta_\lambda - \beta_{\lambda o})^2 - E_{INn}^\lambda\right]\Phi_n^{IN}(\beta_\lambda) = 0. \tag{9}$$

The I_k are the body-fixed angular momentum operators while the reduced moments of inertia of eq. (8) are defined by

$$\mathcal{Q}_k^{(\lambda)}(B_\lambda, \sigma_{\lambda\mu}) \equiv 4B_\lambda\beta_\lambda\, i_k^\lambda(\sigma_{\lambda\mu})$$

and the functional forms are known for the cases $\lambda=2$[14]) and $\lambda=3$[15]). The solutions to eq. (8) have also been given for these two cases[16,10]).

The particular vibrational potential used in eq. (9) has been justified from binding energy data[17]) and need not be discussed further here. By expanding the sum of this term and the previous one about the new equilibrium position $\beta_\lambda(I,N)$, keeping only the quadratic term for the new equivalent potential and defining new independent and dependent variables by

$$y = Z_1 \left[\beta_\lambda - \beta_\lambda(IN)\right] / \beta_\lambda(IN),$$

$$N_\upsilon \, D_\upsilon(\sqrt{2}\, y) = \beta_\lambda^{3/2} \, \Phi_n^{IN} (\beta_\lambda),$$

N_υ a normalization constant, eq. (9) can be placed in the form

$$\left[\frac{d^2}{dy^2} + (2\upsilon + 1 - y^2)\right] D_\upsilon(\sqrt{2}\, y) = 0$$

which is Weber's equation for parabolic cylinder functions[18]). The quantity υ is determined by the boundary conditions

$$\Phi_n^{IN} (\beta_\lambda = 0) \neq 0$$

$$\lim_{\beta_\lambda \to \infty} \Phi_n^{I,N} (\beta_\lambda) = 0$$

while the Z_i are known functions of υ[10].

Calling I_υ the integral

$$I_\upsilon = \int_{-Z_1}^{\infty} D_\upsilon^2 (\sqrt{2}\, x)\, dx$$

the normalization constant, N_υ, can be expressed in the form

$$N_\upsilon^2 = Z_1/\mu\beta_{\lambda o} Z I_\upsilon$$

where μ is the stiffness parameter and Z is the positive real root of

$$Z^4 - (1/\mu)\, Z^3 - \tfrac{1}{2}\,(\epsilon_{IN}{}^{\lambda} + \tfrac{3}{2}) = 0$$

and is related to Z_1 by

$$Z_1^4 = Z^4 + \tfrac{3}{2}\,(\epsilon_{IN}{}^{\lambda} + \tfrac{3}{2}).$$

2. Electric Monopole Transitions

The absolute transition probability for electric monopole conversion has been defined by Church and Weneser[19] as

$$T(E0) = \Omega\, \rho^2$$

where the electronic factor, Ω, is available in graphic form[19] while the nuclear strength parameter is

$$\rho = \Sigma_p \int \phi_f^* \left[\left(\frac{r_p}{R}\right)^2 - \sigma \left(\frac{r_p}{R}\right)^4 + \dots \right] \phi_i \, d\tau \tag{10}$$

the ϕ being the nuclear wave functions, r_p the radius vector of the p^{th} proton, R the nuclear radius, and σ is a numerical coefficient of the order of 0.1. In collective model calculations it is customary to neglect all but the first term of eq. (10). Using the assumption of a uniform charge distribution one can define an electric monopole operator $\mathcal{O}(E0)$ such that

$$\rho = \langle \Phi_{n'}^{IN'}(\beta_\lambda) | \mathcal{O}(E0) | \Phi_{n'}^{IN'}(\beta_\lambda) \rangle . \tag{11}$$

Then

$$\mathcal{O}(E0) = \frac{3Z}{4\pi} \left(\frac{4\pi}{5} + \beta_\lambda^{\ 2} + \frac{5}{3} \beta_\lambda^{\ 3} T_\lambda \right), \tag{12}$$

Z the atomic number. The first term in eq. (12) makes no contribution to ρ, the second term induces transitions between "beta" bands and ground state bands while the third term induces transitions between the gamma and ground state bands for $\lambda=2$. For $\lambda=3$ this term is identically zero so that no E0 transitions can occur between the octupole analog of the gamma band (sometimes called the "g" - band[20])

and the octupole ground state band, e.g., in the notation of ref. [15]) the model permits no E0 transitions between the 311- and 321- levels, even if asymmetry vibrations are included.

Since we are going to consider monopole transitions between "beta" and ground state bands only the term proportional to β_λ^2 in eq. (12) will be retained. Thus eq. (11) can be written

$$\rho^2 = \left(\frac{3Z}{4\pi}\right)^2 \left(\frac{\mu Z\beta_{\lambda d}}{Z_1}\right)^4 \frac{I^2_{\upsilon_f\upsilon_i}(o)}{I_{\upsilon_i} I_{\upsilon_f}} . \tag{13}$$

where the overlap integral $I_{\upsilon_f\upsilon_i}(o)$ is defined by

$$I_{\upsilon_f\upsilon_i}(o) = \int_0^\infty D_{\upsilon_f}\left(\sqrt{2}\left[x-z_1\right]\right) D_{\upsilon_i}\left(\sqrt{2}\left[x-z_1\right]\right) x^2 dx \tag{14}$$

which is identical in form with the overlap integrals arising from the vibrational contributions to the electric quadrupole reduced matrix elements between states of positive parity[4]).

In fig. 1 ρ^2 is plotted for transitions between the positive parity beta band and the positive parity ground state band for I = 0,2,4,6 and 8 as a function of the stiffness parameter μ. The asymmetry parameter has been taken as $\gamma = 0^\circ$, i.e., axial symmetry; however, the curves for other values of γ are only slight different from those given here. In fig. 2 we have plotted ρ^2 as a function of

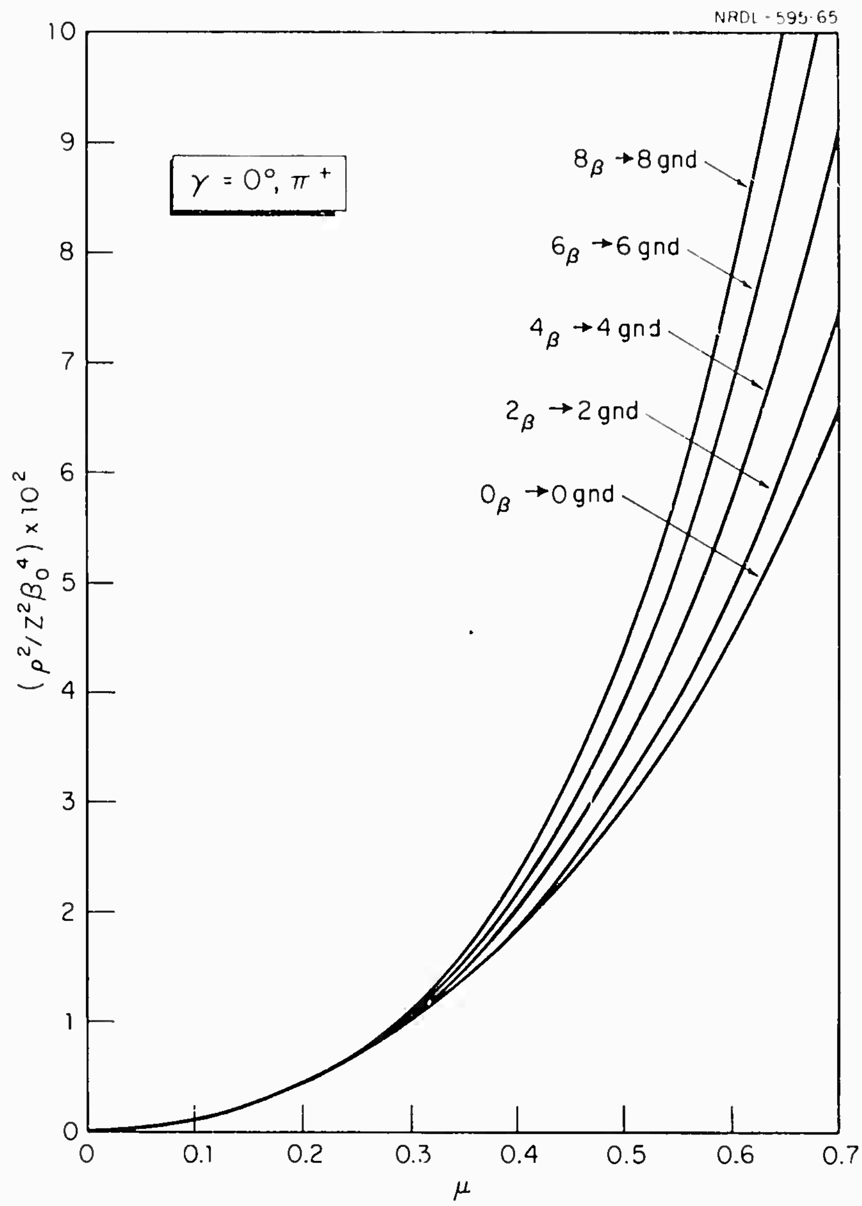

Fig. 1 The monopole nuclear strength parameter $\rho^2/Z^2\beta_o^4$ of eq. (13) plotted as a function of the stiffness parameter μ for an axially symmetric system ($\gamma = 0^o$). The curves are labeled with the spin of initial and final state and are transitions from the beta band to the ground state band for quadrupole deformations. Here Z is the atomic number and β_o the equilibrium deformation.

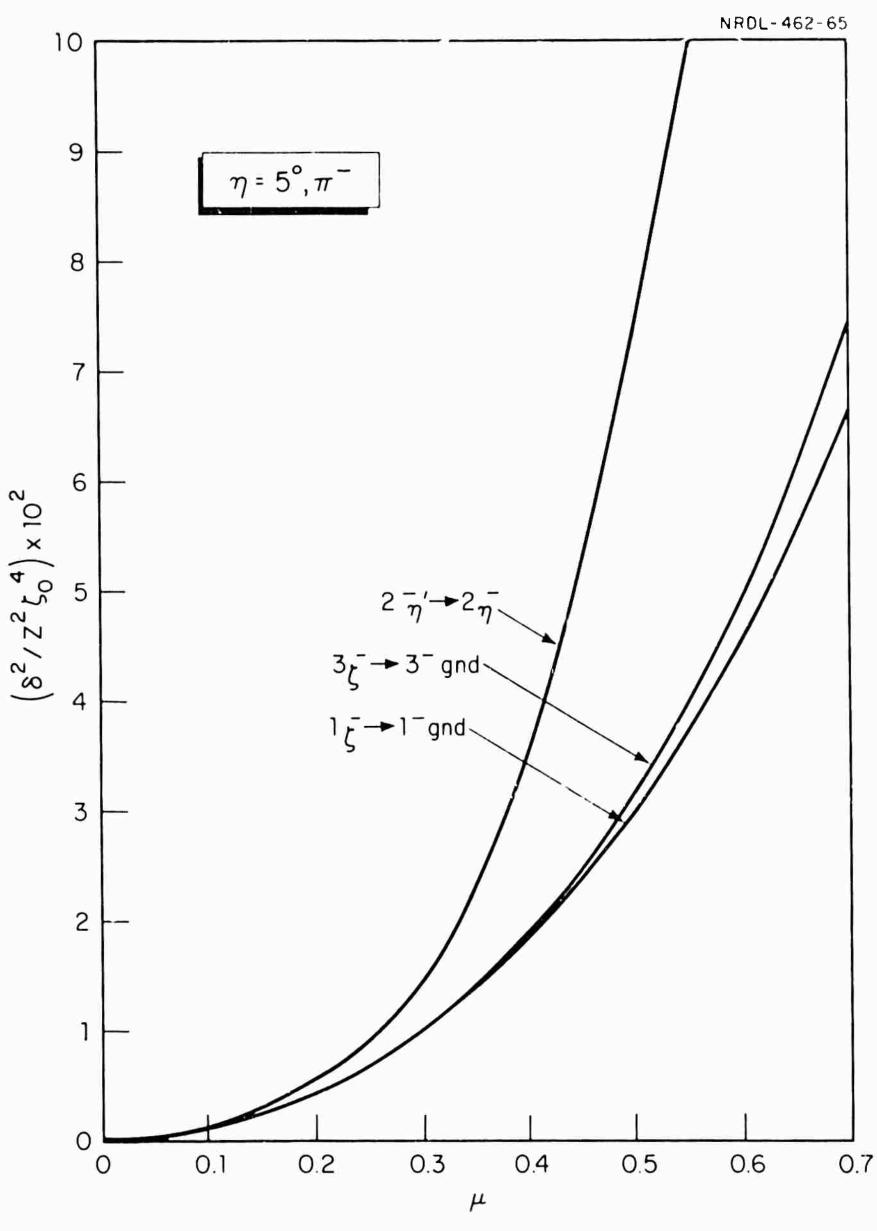

Fig. 2 The monopole nuclear strength parameter $\rho^2/Z^2\zeta_o^4$ plotted as a function of the stiffness parameter μ for an almost axially symmetric octupole system ($\eta = 5^o$). The curves are labeled with the initial and final state spin and are for transitions from the "b-band" to the octupole ground state. Here Z is the atomic number and ζ_o the octupole equilibrium deformation.

μ for monopole transitions between the ζ band and the ground state octupole band for I = 1,2 and 3. The octupole asymmetry parameter has been taken as $\eta = 5^{\circ}$ since for $\eta = 0^{\circ}$ the 2- level lies at infinitely high energy. These curves do show more variation with asymmetry parameter than do those for transitions between positive parity states.

The quantity ρ^2 is not the most useful one to compare with experiment. Frequently one compares the relative rates of E0 and the competing E2 transitions defined by[21])

$$\mu_K(I_i \rightarrow I_f) = \frac{T(E0:\ I \rightarrow I)}{T(E2:\ I_i \rightarrow I_f)} . \tag{15}$$

A somewhat more useful quantity is the dimensionless ratio defined by Rasmussen[12]) for transitions between the beta band I = 0 level and the ground state band as

$$X(0_\beta+ \rightarrow 0_{gnd}+) = \frac{\rho^2 e^2 R_o^{\ 4}}{B(E2:\ 0_\beta \rightarrow 2_{gnd})} \tag{16a}$$

where $R_o = 1.2\ A^{1/3} \times 10^{-13}$ cm is the nuclear radius. For transitions other than from the beta band head we may generalize this quantity to

$$X(I_\beta+ \rightarrow I_{gnd}+) = \frac{\rho^2 e^2 R_o^{\ 4}}{B(E2:\ I_\beta \rightarrow I_{gnd})} \tag{16b}$$

of course $I_\beta = I_{gnd} \neq 0$. The relation between X and μ_K is just

$$X = 2.53 \frac{\mu_K A^{4/3} E_\gamma^5}{\Omega} \times 10^9 \tag{17}$$

where the gamma transition energy E_γ is in MeV.

By making use of the expression for the reduced E2 transition probabilities developed elsewhere[4]) these expressions for the dimensionless ratio X can be expressed in terms of overlap integrals (14) and the other parameters of the theory. For transitions from the beta band head eq. (16a) becomes

$$X(0_\beta + \rightarrow 0_{gnd}+) = (\mu\beta_{20})^2 \, (Z_i/Z_{1\ i})^5 \, (Z_{1\ f}/Z_f)^3 \, I_{\upsilon_f}$$
$$\times \, I_{\upsilon_i \upsilon_f}(0)^2 / I_{\upsilon_f} \, I_{\upsilon_i \upsilon_f}(2)^2 \, b(E2{:}\ \ 0\ 1\ + \rightarrow 21+) \tag{18a}$$

while for transitions from other beta band levels eq. (16b) may be written

$$X(I_\beta + \rightarrow I_{gnd}+) = (\mu\ \beta_{20}\ Z_i/Z_{1\ i})^2$$
$$\times \, I_{\upsilon_i \upsilon_f}(0)^2 / I_{\upsilon_i \upsilon_f}(2)^2 \ \ b(E2{:}\ \ I_\beta\ N_\beta + \rightarrow I_g\ N_g+). \tag{18b}$$

In eqs. (18a,b) the subscripts i and f refer as usual to initial and final states while the notation (0) and (2) on the overlap integrals

refer to the integral of eq. (14) for monopole transitions or a similar one for E2 transitions[4]). (For the latter the functional dependence is different since Z and Z_1 are different in initial and final states.) The quantity $b(E2:\ IN \rightarrow I'N')$ is defined from the adiabatic reduced transition probability of ref.[4]) by

$$B_a(E2:\ \ IN \rightarrow I'N') = \left(\frac{3Z\ e\ R_o^2}{4\pi}\right)^2\ b(E2:\ \ IN \rightarrow I'N')$$

Ze being the nuclear charge.

In fig. 3 the ratio $X(0_\beta \rightarrow 0_{gnd})/\beta_{20}^2$ is plotted as a function of the asymmetry parameter, γ, for various values of μ. In figs. 4 and 5 the ratio $X(I_\beta \rightarrow I_{gnd})/\beta_{20}^2$ are plotted as a function of γ again for various values of μ. Fig. 4 is for the 2+ → 2+ transition while fig. 5 is for the 4+ → 4+ transition. Examples of both have been measured.

Finally, for monopole transitions between negative parity levels the ratio $X(I_\zeta \rightarrow I_{gnd})$ is especially simple since the E0 and E2 overlap integrals are identical and thus cancel. Thus, eq. (16b) can be written for such transitions as

$$X(I_\zeta- \rightarrow I_{gnd}-) = 3\pi(\mu\ \zeta_o\ Z/Z_1)^2/4b(E2:\ \ I_\zeta N_\zeta- \rightarrow I_{gnd}\ N_{gnd}-). \qquad (19)$$

The quantity $X(I_\zeta- \rightarrow I_{gnd}-)/\zeta_o^2$, where $\zeta_o = \beta_{30}$, is plotted in fig. 6 as a function of the octupole asymmetry parameter, η, for several

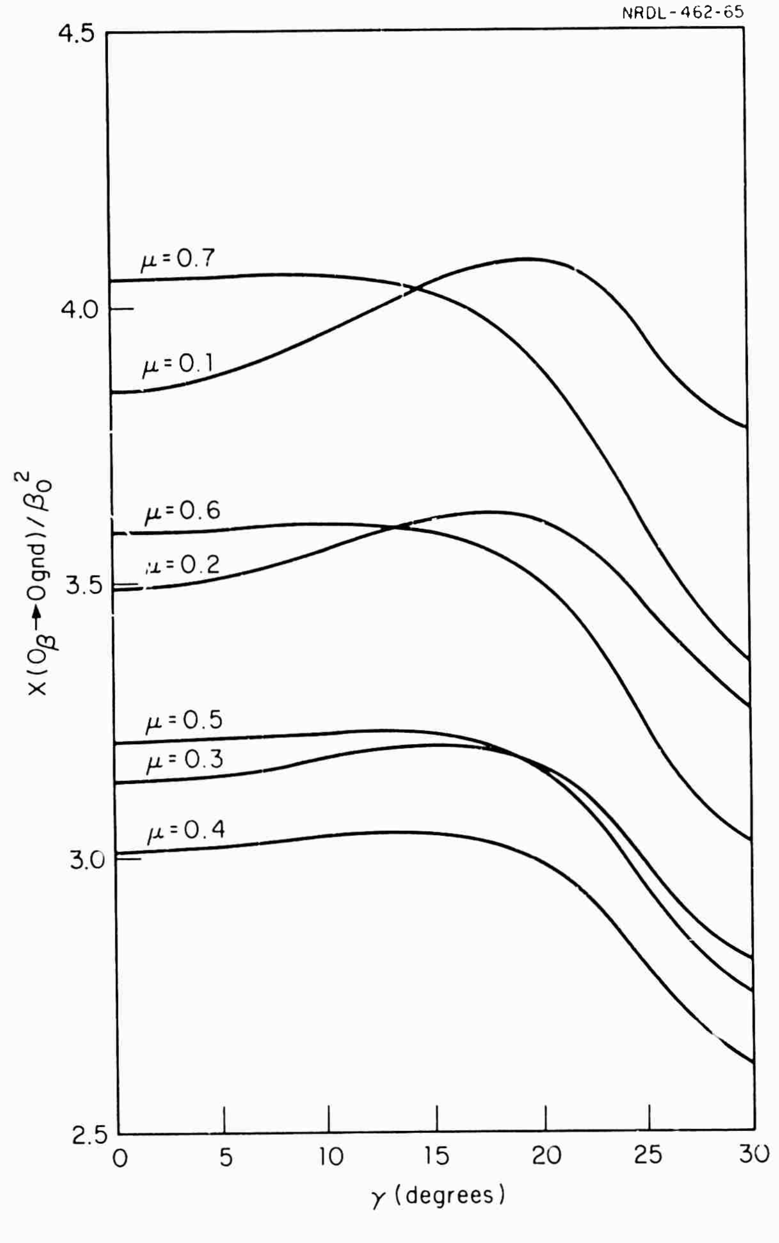

Fig. 3 The dimensionless ratio $X(0_\beta \rightarrow 0_{gnd})/\beta_o^2$ plotted as a function of the quadrupole deformation parameter, γ, for several values of the stiffness parameter μ.

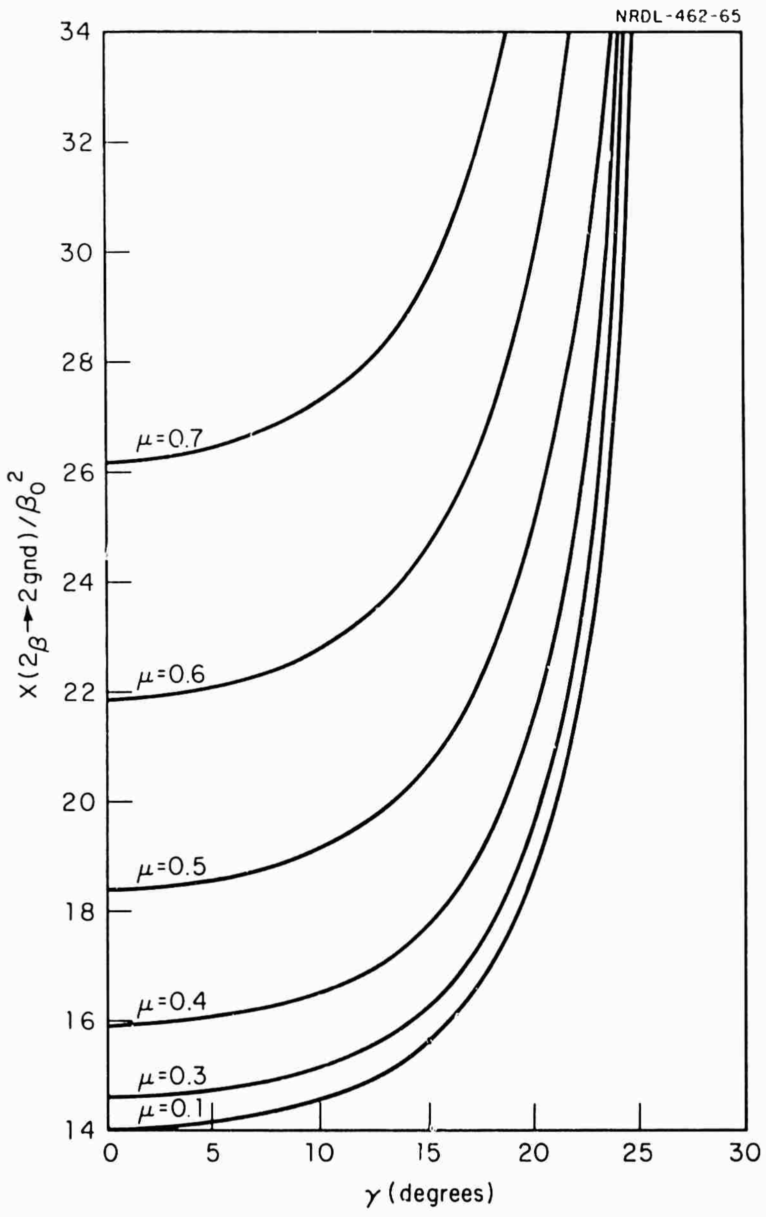

Fig. 4 The dimensionless ratio $X(2_\beta \rightarrow 2_{gnd})/\beta_o^2$ plotted as a function of the quadrupole deformation parameter, γ, for several values of the stiffness parameter μ.

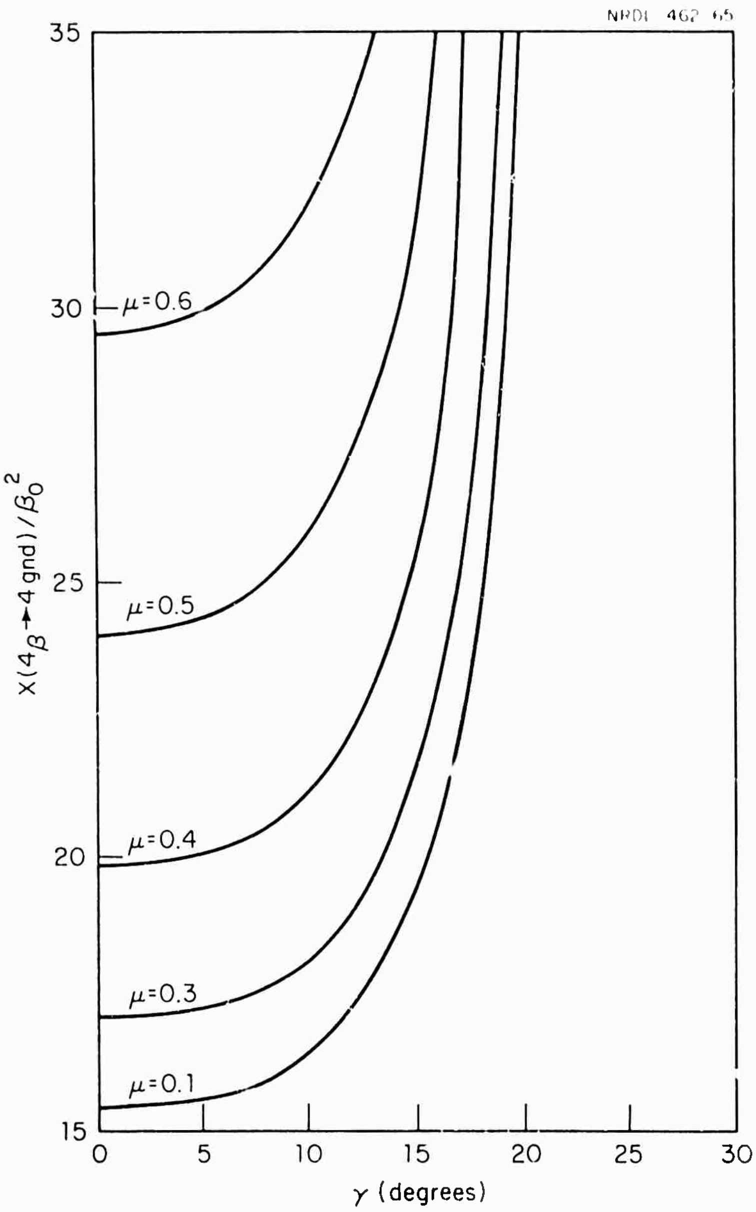

Fig. 5 The dimensionless ratio $X(4_{\beta} \rightarrow 4_{gnd})/\beta_o^{\ 2}$ plotted as a function of the quadrupole deformation parameter, γ, for several values of the stiffness parameter μ.

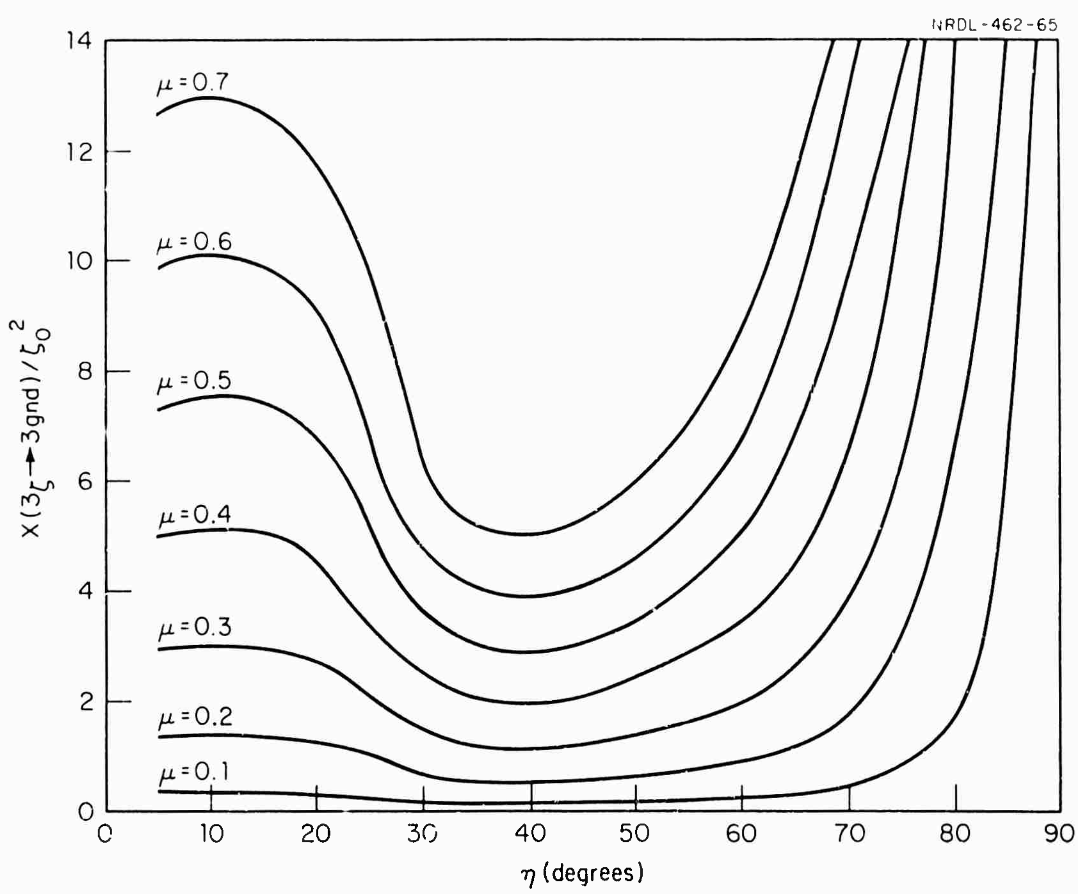

Fig. 6 The dimensionless ratio $X(3_{\zeta}- \rightarrow 3_{gnd}-)/\zeta_o^2$ plotted as a function of the octupole deformation parameter, η, for several values of the stiffness parameter μ. This ratio is for transitions between the lowest 3- state in the octupole "b" band to the lowest 3- state in the octupole ground state band.

value of μ for the case $I = 3-$.

3. Comparison with Experiment

In table 1 we compare this calculation with the available experimental results for E0 transitions from various levels of the beta band to the ground state band both for rare earth deformed nuclei and the actinides. In columns 2 and 3 are given the values of γ and μ which reduce the r.m.s. deviation between theory and experiment of the energy level structure to a minimum. In particular it has been found that the values of μ are high where a deformed region opens and decrease rapidly to about 0.2. This value is maintained into the transitional region where μ begins a slow increase. The best fit values of μ seem to be correlated with the number of neutrons beyond the nearest closed shell and are independent of the deformed region in which the nuclide in question is found[6]). The fourth column gives the equilibrium deformations $\beta_o = \beta_{20}$ which have been fit to the reduced transition probability for coulomb excitation to the first excited ($I\pi = 2+$) state[4])

$$B(E2:\ 011 \rightarrow 211) = \left(\frac{3ze\beta_o \mu R_o^2}{4\pi}\right)^2 b(E2:\ 01 \rightarrow 21)$$
$$\times \left(Z_{1i}/Z_i I_{\upsilon_f} I_{\upsilon_i}\right)\left(Z_f/Z_{1f}\right)^3 \ I_{\upsilon_i\,\upsilon_f}(2)^2. \qquad (20)$$

Where the coulomb excitation data is either not available or of

Table 1

Nucleus	γ	μ	β_o	E_t(keV)	I	X/β_o^2 Thy	X/β_o^2 Exp.	Ref.
^{152}Sm	10.73	0.3996	0.314	685	0	3.40	0.88 ± 0.07	22)
				689	2	16.6	8.7	23)
^{154}Gd	11.62	0.4006	0.303	681	0	3.42	0.84	24)
^{156}Gd	10.04	0.2735	0.342	1010	0	3.30	6.8	25)
^{158}Dy	11.81	0.2916	0.335	986	2	15.8	4.61	26)
^{164}Er	12.39	0.2509	0.329	1217	2	15.6	1.35	26)
^{168}Yb	10.85	0.2561	0.325	1104	4	18.0	0.731	26)
^{178}Hf	9.750	0.2614	0.281	1197	0	3.33	2.3 ± 0.5	27)
				1440	0	3.33	6.7 ± 2.0	27)
^{184}Pt	12.43	0.5192	0.198	677	2	20.45	3.36	9)
				796	4	28.75	2.16	9)
^{230}Th	9.375	0.2714	0.267	634	0	3.25	3.1 ± 1.4	28)
^{232}Th	9.049	0.2464	0.272	730	0	3.40	4.6 ± 1.1	28)
^{232}U	8.365	0.2482	0.279	693	0	3.39	2.2 ± 0.5	28)
^{234}U	8.619	0.2201	0.285	811	0	3.48	6.2 ± 1.0	28)
^{238}U	7.914	0.2030	0.297	993	0	3.54	2.4 ± 0.6	28)
^{238}U	7.898	0.2068	0.281	941	0	3.53	8.0 ± 2.5	28)
^{240}Pu	8.345	0.2131	0.283	870	0	3.49	0.62 ± 0.12	28)

Comparison of experimental values of electric monopole transition probabilities with theory for transitions from the beta to the ground state band. In column 1 are the nucleus and A value, columns 2, 3, and 4 give the best fit values to the quadrupole asymmetry parameter, stiffness parameter and equilibrium deformation parameter. Column 6 is the energy of the monopole transition, column 7 the spin of the levels involved while columns 8 and 9 give the theoretical value and experimental value, with errors where quoted, of the dimensionless ratio X/β_o^2. The quantity X is defined in eq. (16a) for $0 \rightarrow 0$ transitions and in eq. (16b) for all other E0 transitions.

insufficient accuracy use was made of Grodzins' analysis of the E2 transition probabilities from which one can obtain the empirical relation[29])

$$\beta_o = 1108/A^{7/6} E_\gamma^{1/2} \tag{21}$$

E_γ the energy of the first excited state in keV. This relation is quite accurate in the deformed regions and the values of β_o given by eqs. (20) and (21) are very close.

Table 1 shows that except for ^{178}Hf, ^{156}Gd and the $2_\beta \rightarrow 2_{gnd}$ transition in ^{152}Sm the monopole transitions in the rare earth deformed nuclei are from five to ten times smaller than predicted by theory. (The experimental value for the $0_\beta \rightarrow 0_{gnd}$ transition in ^{152}Sm is that reported in ref. [22]). Rasmussen[12]) making use of some unpublished data of the Chalk River group takes half of this value which clearly leads to a no more favorable comparison.) On the other hand for the actinide nuclei agreement between experiment and theory is quite satisfactory except for the nucleus ^{240}Pu. However, Bjørnholm[28]) has pointed out that in this case the very low value may be associated with the fact that the beta band head lies very close to the neutron energy gap. One might expect that this would offer an explanation for the failure of the theory in the rare earth region, and certainly in the middle of the region the beta band heads are rather close to the gap.

However, this can hardly account for the lack of agreement for the $0_\beta \rightarrow 0_{gnd}$ transition in ^{152}Sm where the band head at 685 keV is well below the gap. Furthermore, for the $2_\beta \rightarrow 2_{gnd}$ transition the comparison is adequate especially in view of the fact that the B(E2) from the beta band is somewhat smaller than predicted by theory[4]). It would seem quite worthwhile for both of these monopole transitions in ^{152}Sm to be measured by the same group to see if this discrepancy persists.

Several of the rare earth nuclei listed in the table are known to have more than one 0+ excited state and for ^{178}Hf the X ratios for two such levels have been measured. In this nucleus we have taken the lower 0+ level at 1197 keV as the beta band head for which the μ value is consistent with the general trend in the region. The X/β_o^2 ratio for the 1400 level has been calculated assuming that it too is a 012 level. It very probably is not the 013 level which the model would predict for these parameters in the neighborhood of 2 MeV. This level must then have a different character from that of a 0+ beta vibrational level -- perhaps it is a two quasi particle state. No attempt has been made to calculate the irteraction of these two states and it seems hardly worthwhile to construct a theory of higher excited 0+ states until more are known and their characteristics determined. A true beta band should not only have enhanced reduced E2 matrix elements to the ground state band but should as well have large E0 transition probabilities. Thus investigation of the excited 0+ levels must involve not only the

determination of E2 strengths but E0 strengths as well.

It has been suggested that the failure of the theory in the rare earth region might be due to the influence of a non-uniform charge distribution, which was assumed in order to derive the operator $\mathcal{O}(E0)$ of eq. (12) or of the lack of irrotational nuclear flow[21]). It is doubtful if any but the most drastic innovation would induce the needed order of magnitude change in X and this in turn would influence greatly, no doubt unfavorably, the agreement already obtained for the E2 transitions[4,6]). Also it is difficult to believe that such a change in the theory would not destroy the agreement for the E0 transitions so evident in the actinide region. In any event, we should like to see many more measurements of the monopole transition probability in the rare earth region especially for transitions other than $0_\beta \rightarrow 0_{gnd}$. A particularly good candidate is ^{150}Nd whose level structure seems quite analogous with ^{152}Sm.

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Reduced transition probabilities for electric monopole transitions ρ^2 have been calculated using a collective model with deformation vibrations but no asymmetry vibrations. Both ρ^2 and $X = \rho^2 R_0^4/B(E2)$ have been evaluated within the context of the model by exact numerical methods for transitions within positive as well as negative parity rotational bands of even nuclei. Comparison of experiment with theory shows quite good agreement in the actinide deformed region; however, for the rare earths theoretical predictions are generally an order of magnitude greater than the measured values.

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